Rayleigh Damping

The n x n symmetric damping matrix [C] is formulated as a linear combination of the mass [M] and stiffness [K] matrices:

$[C] = \alpha[M] + \beta[K]$ (Equation 1)

- **Alpha Coefficient:** Sets the mass-proportional coefficient α.
- **Beta Coefficient:** Sets the stiffness-proportional coefficient β.

The type of damping described by (Equation 1) is known as Rayleigh or proportional damping.

This form of [C] is orthogonal with respect to the system eigenvectors.

By applying the modal coordinate transformation, the modal damping matrix [c] becomes diagonal:

$[\Phi]^T[C][\Phi] = [c] = \alpha[1] + \beta[\omega^2]$ (Equation 2)

You can define Rayleigh damping for linear and nonlinear dynamic studies.

Relation of Rayleigh Coefficients and Modal Damping Ratio

The modal damping matrix [c] is given by:

$[c] = 2[\zeta\omega]$ (Equation 3)

The coefficient of viscous damping c_i for the ith mode is calculated by:

$c_i = 2\zeta_i\omega_i = \alpha + \beta\omega_i^2$ (Equation 4),

and the viscous damping ratio ζ_i is expressed as

$\zeta_i = \alpha / (2\omega_i) + \beta\omega_i / 2$ (Equation 5)

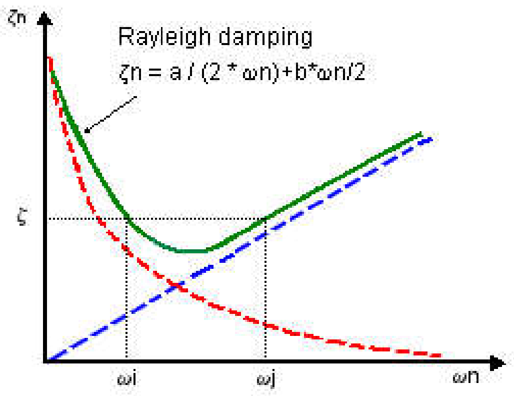
If the damping ratios for the ith and jth modes are ζ_i and ζ_j, then the Rayleigh coefficients α and β are calculated from the solution of the two algebraic equations:

$$\frac{1}{2}\begin{bmatrix} 1/\omega_i & \omega_i \\ 1/\omega_j & \omega_j \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \begin{Bmatrix} \zeta_i \\ \zeta_j \end{Bmatrix}$$ (Equation 6)

If both modes have the same damping ratio ($\zeta_i = \zeta_j = \zeta$) , then the values of α and β are given by:

$$\alpha = \zeta\frac{2\omega_i\omega_j}{\omega_i + \omega_j}$$ (Equation 7) $$\beta = \zeta\frac{2}{\omega_i + \omega_j}$$ (Equation 8)

The viscous damping ratio ζ for any other mode varies with frequency as shown in the figure:



Parent Topic

Damping Effects

Related Concepts

Modal Damping

Composite Modal Damping

Related Reference

Global Damping PropertyManager

Spring-Damper Connector